The Use of Pivot Point in Ship Handling
for Safer and More Accurate Ship Manoeuvring

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Abstract

The size of ships has increased notably over recent decades. The size of harbours and ports has however not grown in proportion. As a result ship manoeuvring in harbours and ports has become more problematic, and more of an art than a science. The 'pivot point' concept can be useful in analysing slow ship manoeuvring and has therefore been widely adopted by practitioners and training institutions. As a result many practitioners now routinely plan confined manoeuvring using the 'pivot point' concept. Traditionally the 'pivot point' concept has been defined in a number of contradictory and inaccurate ways leading to confusion and mystification. As a result many practitioners and trainers often rely on intuition to bridge the gap between reality and their flawed understanding of theory. In this presentation the theoretical aspect of the pivot point is reviewed and correct definitions put forward and applied to basic and 'special' ship's manoeuvres.

1. Introduction

Traditionally the pivot point of a ship has been defined as the centre of ship's rotation. Thus the ship's motion has become to the eyes of ship handlers a simple one of surge and yaw only. Due to this simplicity, the concept has been very useful in helping to analyse the manoeuvring of a ship, and thus the term is used extensively in teaching and training ship handlers their essential techniques.

However, ship's motion in a small confined area is at least a general planar motion involving surge, sway and yaw. Thus using the traditional definition, calculating the position of the pivot point was not possible, and taken roughly as located at a third (quarter) of ship length from the bow (stern) when moving ahead (astern).

In recent decades ships have become bigger in size and forced to operate in relatively smaller port or harbour areas, which demands more precise and skilled manoeuvring from ship handlers. This in turn made it necessary to know more precisely the location of the pivot point.

In this article the pivot point is regarded as an **apparent centre of rotation,** taking the centre point at midships as the actual centre of ship's yaw motion. A few basic manoeuvres of ships are described in the light of this new definition and some frequently encountered manoeuvres which were difficult to explain using the traditional pivot point definition are also discussed.

2. The Pivot Point

2.1 How is it brought about?

Ship's motion in 3-D space has six degrees of freedom, namely, surge, sway, heave, roll, pitch and yaw. The slow motion of a ship carefully handled in a small confined area, however,

could be modelled as a General Planar Motion (in 2-D plane) for practical purposes, in which only two translations (Surge and Sway) and one rotation (Yaw) are considered.

Each elemental motion is linear in nature, and thus can be superimposed to give the combined effect. For the purpose of analysis, a ship's motion needs to be decomposed into the elemental motions and each element is treated individually. And then all of the elemental motions are combined together to give the resultant motion.

It is usual for mariners to feel ship's movement relative to the surrounding, that is, water surface or fixed objects such as land or buildings - giving rise to the terms *Motion through Water* and *Motion over Ground*. In the following discussion, the water is assumed to be stationary.

When a ship is following a curved path, surging, drifting and turning will happen simultaneously at every instance. In Fig.1 the drifting (black hull) and the turning (blue hull) motions are shown individually. As the two kinds of motion occur simultaneously, a unique point is noted. The point P, where the two kinds of motion cancel each other, will remain stationary, **whereas the turning motion (yaw) itself happened about the point S**. With the surge motion added, the point P will have only the forward motion, giving an observer on board the illusion that the ship is pivoting about this point - thus the name **Pivot Point.**
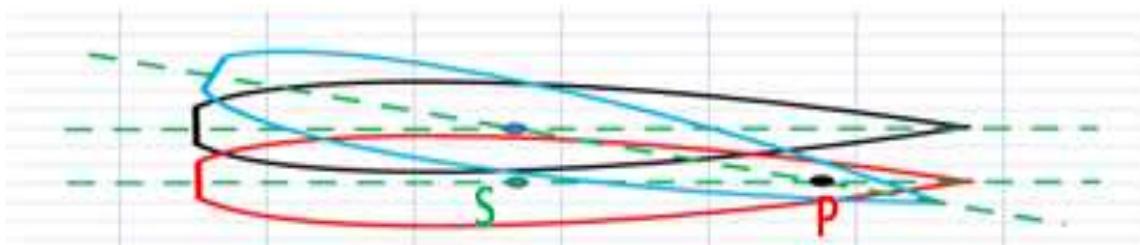

Fig.1 The Birth of Pivot Point

Taking the point S as the actual centre of rotation (yaw motion) and the pivot point P as the point on the ships centre plane at which the displacement due to drift and yaw cancel each other, Eq.1 is established.

$$\mathbf{V} + (\mathbf{X_p} \times \mathbf{r}) = 0 \qquad\qquad [Eq.1]$$

where, $\mathbf{V}$(m/s): sway speed of S

P: Pivot Point

$\mathbf{X_p}$(m): distance to P from S

$\mathbf{r}$(rad/s):yaw Speed

A geometrical observation Fig.2 would now allow a few descriptive definitions.

-The pivot point is the point on a ship's centreline which *appears* to be the centre of rotation to the observer on board ship. (*Apparent* **Centre of Rotation**)

-The pivot point is a point on a ship's centreline which gives the **shortest turning circle radius**.

-The pivot point is a point on a ship's centreline at which the **drift angle is zero**.

-The pivot point is a point on a ship's centreline whose **motion vector** is in line with the ship's instantaneous heading.
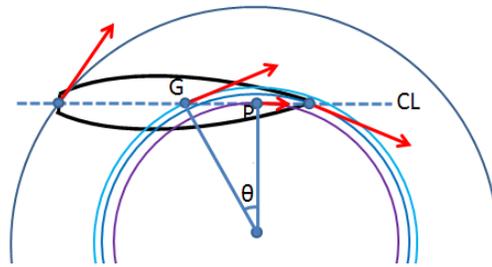
Fig.2 Local Drift Angle, Pivot Point

In Fig.2, the four points shown on the centerline of the ship are the bow, the pivot point, the centre of gravity and the stern. The red arrows show the **direction** of the motion vector of each point. (The magnitudes are not shown.) The angles between the motion vectors and the ship's centerline are called the *drift angles*. As shown in the diagram, the circle passing through the pivot point has the shortest radius.

2.2 How does the pivot point move?

From Eq.1, we obtain,

$$\mathbf{X_p} = \mathbf{-V/r} \qquad\qquad\qquad [Eq.2]$$

This expression implies a few aspects of the pivot point.
- The pivot point cannot be defined without a rotational motion.
- If a rotational motion precedes the sway motion, the pivot point first appears at the point S and converges to a point (P) as the ship approaches a steady state. This can happen when the rudder is first set at an angle to the ship's centreline. An example is seen in Fig.3 which is from Tseng (1998). In this case of Mariner Class vessel, ship trial data testifies that the turning moment the force of which is far away from the centre of gravity of ship, takes effect prior to the sway motion in which the inertial force and lateral resistance is great when the rudder is set at an angle to the centreline. In Fig.3, the graph shows the distance of the pivot point from the bow as a fraction of ship's length. The pivot point is shown to gradually converge from the midship to a location about 13% of ship length from the bow.
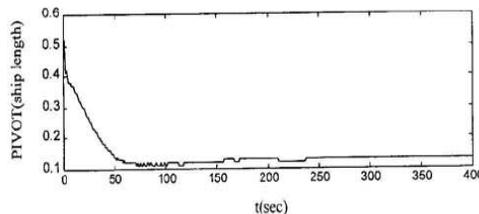

Fig.3 Pivot Point starting from S

-If the sway motion precedes the rotational motion, the pivot point first appears at infinity and very quickly, but decelerating, converges to a point (P). This happens every time the rudder changes the side from starboard to port or vice versa. Essentially the same phenomena are seen in trial data (zigzag run) as shown in Fig.4. (Hwang 1980) This is expected as there will be a point in time when the ship has zero yaw speed while swaying, when she changes the direction of turning from starboard to port or vice versa. In Fig.4 the spikes indicate that the pivot point was momentarily at infinity and the horizontal part of the line shows the steady position of the pivot point. The spikes alternate the direction, first downwards and then upwards, which means first it disappears into the forward infinity and then reappears at aft infinity before quickly returns, with deceleration, to .the steady location.
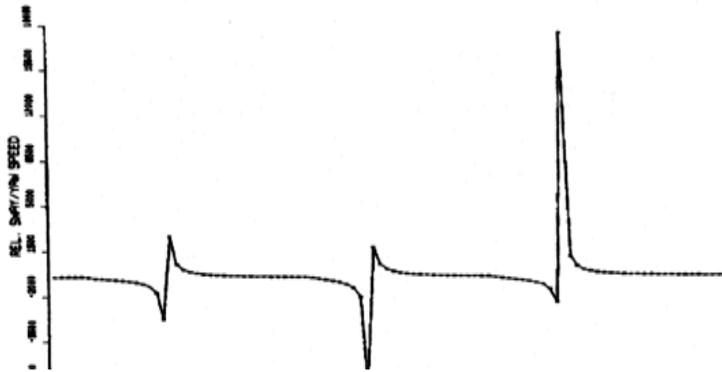
Fig.4 Pivot Point from Infinity

3. What movement results by combining the elemental motions?

When all the three motions (surge, sway, yaw) are present, there exist three distinctive points:-

-the **Centre of Circling** (the Earth-fixed Centre of Planar Rotation: **E**) This is the instantaneous centre of curvature of ship's path.

-the **Centre of Rotation** (the Ship-attached Centre of Yaw motion: **S**) This is the centre of bodily rotation (spinning), for which it is usual to take the centre of gravity of ship's mass and added mass in ship dynamics calculation.

-the **Pivot Point** (the Apparent Centre of Rotation: **P**) This is the shadow of point E projected on to the ship's centreline. (Naukowe 2010)

3.1 Ship Motion with Yaw but No Translation

This is the case when the ship is turning about its own centre of rotation (S). The ship is not moving ahead/astern, nor is swaying. In this case all three points coincide. (Fig.5) This manoeuvre could be produced with the bow thruster and stern thruster in opposite direction.
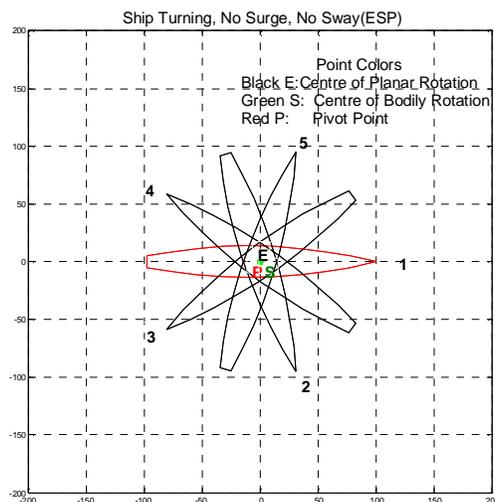

Fig.5 Yaw only

3.2 Ship Motion with Yaw and Sway

In the absence of any longitudinal movement (no surge), if the ship drifts at the same time as turning, and if the pivot point is between S and the bow, the motion shown in Fig.6 will results. In this case the two points, E and P, will coincide.

Ship Turning and Drifting, No Surge, PP between Midship and Bow (ESeP)

Black E: Centre of Planar Rotation
Green S: Centre of Bodily Rotation
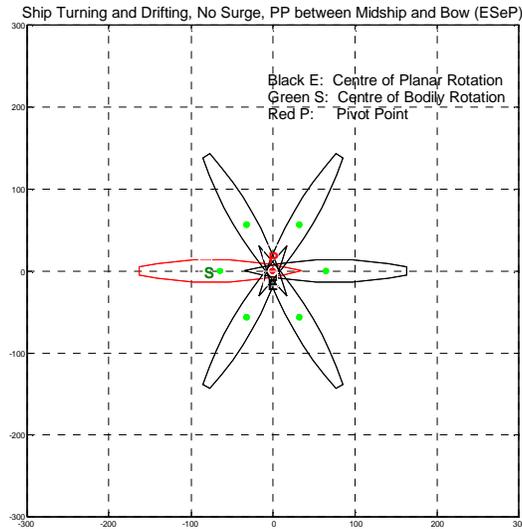Red P: Pivot Point

Fig.6 Yaw+Sway

If the pivot point is ahead of the bow, the motion shown in Fig.7 will result.
The two points, E and P, are at the same location.
This manoeuvre could be produced using the stern thruster. In practice however, the same manoeuvre could be produced by a combination of all the three elemental motions. (See Section 4.2.)
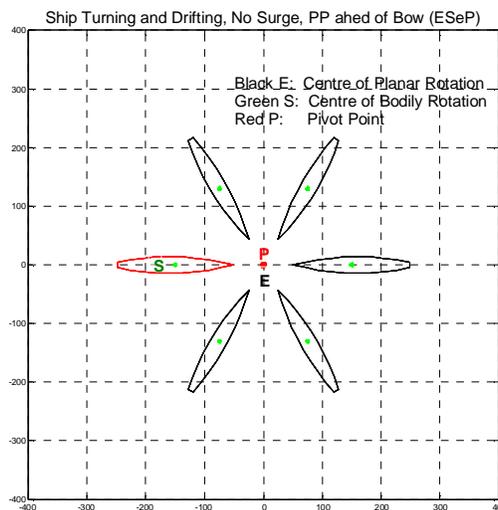
Ship Turning and Drifting, No Surge, PP ahed of Bow (ESeP)

Black E: Centre of Planar Rotation
Green S: Centre of Bodily Rotation
Red P: Pivot Point

Fig.7 Yaw+Sway

3.3 Ship Motion with Yaw and Surge

If the ship moves ahead while turning but without any sway motion, the resulting movement will look like the one shown in Fig.8. In this case, the pivot point will be on top of the point S.

This manoeuvre could be produced with both bow and stern pods deflected. This manoeuvre without any drift causes no swing out of the stern, thus it is suitable in a restricted waters.
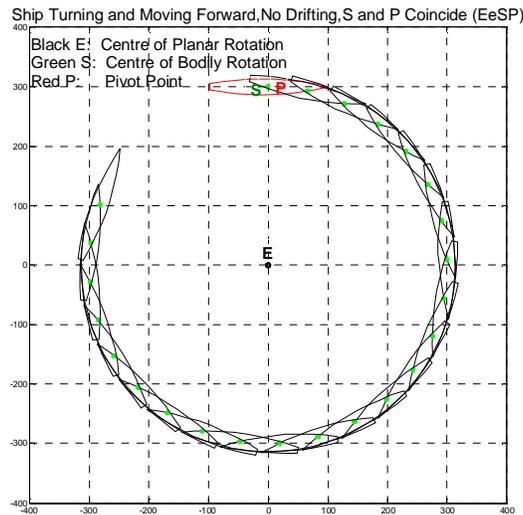

Fig.8 Yaw+Surge

3.4 Ship Motion with Heading, Drifting and Turning

When all the three motions (Surge, Sway, Yaw) are present, all the three distinctive points (E, S, P) will exist separately as shown in Fig.9. In this particular case, the stern swings out sweeping a bigger arc, as all skilful ship handlers are most conscious of.

Ship motions in general fall in this category. The amount of swing out is directly related with the position of the pivot point.
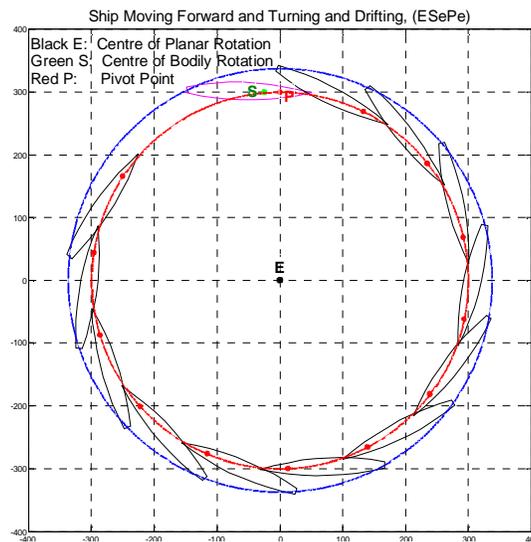

Fig.9 General Planar Motion

4. Applications of the new definition

4.1 Can the Pivot Point be ahead of Bow?

The converged position of Pivot Point could be ahead of the bow, if
-the drift angle of point S, $\theta > \sin^{-1}(L/2R)$
-the turning circle radius $R > (L/2\tan\theta)$
  where θ is the drift angle of S when P is at the bow
- the ship circles with a curvature less than $(2\tan\theta/L)$
Also from Eq.2, $\mathbf{X_p} = \mathbf{-V/r}$, $\mathbf{-V} > 0.5L\mathbf{r}$ for the pivot point to be ahead of bow.

4.2 How to make the ship "walk sideways"?

When the propeller starts to turn with the rudder set to an angle, the ship starts to surge followed by sway and yaw. If the forward motion is controlled to a small amount, and if enough time is allowed for drifting and turning motion to manifest, there may be a combination of motions for the ship to move in a circular path sideways, keeping the pivot point ahead of the bow as the centre of circling. Imagine a ship positioned at the bottom heading upwards in the chart and kick started. After a small time interval, the ship will have moved forward, drifted and turned. If the ship has moved in a circular path sideways, the distance moved forward is the same as the vertical distance of the circular path from a horizontal starting datum line. The combined result would be the same as moving the ship sideways in a circular path without surge motion as shown in Fig.7.

4.3 Docking astern into a cut

Fig.10 shows a possible manoeuvre according to a traditionally given definition of the pivot point which would locate the pivot point somewhere aft of midship (e.g. the Red dot). The ship has been moving astern. The stern is now pushed (by the pod, for example) to turn into the slip. This manoeuvre as it is shown, however, no experienced ship handler would adopt as he would know this will not work unless the manoeuvre is assisted by the bow thruster. According to the new point of view, the pivot point is much further away from the stern which is being pushed, making it necessary to push the bow away (by the bow thruster, for example) from the pier, to dock the stern into the slip safely. (Cauvier 2008)
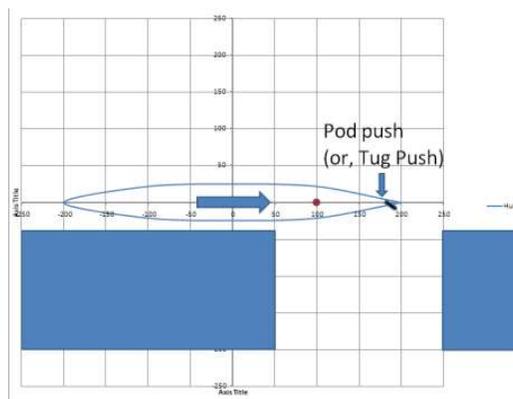

Fig.10: Docking Astern into Slip

4.4 Standing turn from stopped position

Going round a corner with a tightest possible turn is a frequently faced situation, for which experienced mariners would use kick ahead with the rudder put at 35°. This technique is employed according to the traditional description of pivot point characteristics which says

that the pivot point would move instantly to about a third/quarter of the ship's length from the bow, thus hoping to create the motion shown in Fig.9 for the first 90°. However, these attempts often fail to clear the corner even when the ship's stopped position is well ahead of the corner at the starting point so that the imagined pivot point can be ahead of the corner. Those failures imply that the assumptions are erroneous. In reality, the pivot point first appear at midship since the turning motion manifest prior to the drifting motion. And then it moves gradually forward as the drift motion sets in. In the case of Mariner class ship, this takes about 50 seconds as shown in Fig.3. Therefore, the gap between the ship and the peer should be carefully chosen for each ship to use the technique.

5. **Analytical calculation of Pivot Point**

Imagine a box barge with a uniform density floating in water. Assume the water and air exert no resistance on the barge. The centre of gravity (G) will be at the midships. If there exist a constant lateral force (F_R) at the stern (such as rudder force), it will cause the barge to sway and yaw simultaneously. Both of these motions will be uniform acceleration motions. After a small time interval t, the centre of gravity will have moved a short distance GG_1 by the sway motion. (Fig.11) At the same time, the barge will have turned a small angle, θ about the centre of gravity. This angle is shown at P in Fig.11.
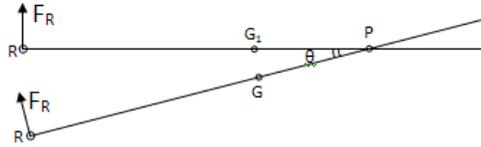

Fig.11 Box Barge Rudder Force applied

Taking the distance between the centre of gravity and the pivot point as GP, the distance GG_1 can be calculated by GP x θ. Thus,

$$GG_1 = GP \times \theta = \frac{1}{2}at^2 = \frac{1}{2}(\frac{F_R}{\Delta})t^2$$ [Eq.3]

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(\frac{T}{I})t^2 = \frac{1}{2}(\frac{F_R \times GR}{I})t^2$$ [Eq.4]

Here the turning moment T = F_R x GR where GR is the distance between the centre of gravity and the point at which the force is applied (e.g. rudder). This is taken as the same as half the ship length.

Dividing Eq.3 by Eq.4 yields,

$$GP = \frac{I}{\Delta \times GR} = \frac{I_z \times d \times \rho}{L \times B \times d \times \rho \times GR} = \frac{I_{XX} + I_{YY}}{L \times B \times GR} = \frac{L^2 + B^2}{12 \times GR}$$ [Eq.5]

Eq.5 affirms that the pivot point is a purely geometrical property. It is neither dependent on the magnitude of the applied force nor on the total weight of the ship.

6. Numerical calculation of pivot point

The mathematical derivation of equations in the previous chapter was only for the idealised case. In reality, it is not that simple. There will be water resistance and air resistance, and the issue of added mass, etc. Also the ship will be trimmed and the viscosity of water will change the flow (pressure field) significantly. Thus analytic solution is not possible, a numerical solution must be employed instead.

The potential flow part can efficiently and accurately be calculated by some Boundary Element Methods. Seo (1984) used Rankine panels on the immersed part of ship's hull and on a portion of the disturbed free surface in the vicinity of the ship, thus the body boundary condition and the free surface boundary condition was satisfied exactly (within the numerical error bound). The unbounded free surface outside the panelled region was also included in the computation by extrapolation using exponential function fit to the free surface panel source strengths.

With the advent of powerful computing engines during the last decade or so, it is now common to attempt to solve the Navier-Stokes equation directly by some numerical schemes. Combined with potential flow calculations, these viscous flow computations produce some fairly realistic predictions of the flow around ships.

Considering all the hydrodynamic and aerodynamic forces and added mass, the sway and yaw velocities may be calculated for the centre of gravity. The position of the pivot point may then be found using Eq.2.

7. Summary and Discussion

The following a few points could be mentioned among others:-
- The pivot point is not the actual centre of ship's yaw motion, but just appears to be so. Nevertheless, it is a useful concept for ship handlers to visualize the movement of ship.
- The pivot point does not move instantly, but rather gradually corresponding to the changing hydrodynamic/aerodynamic surroundings.
- A surge motion alone can not create the pivot point.
- The pivot point is a geometrical property, not a physical quantity.

The complex numerical calculations in ship manoeuvring simulation software includes the computation of the linear sway speed and the angular yaw speed, which will give the pivot point location. In the light of the usefulness of the pivot point to the ship handlers, it is considered to be well worth including the instantaneous pivot point location together with those of the immediate future on a display monitor.

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Symbols and Abbreviations

E	centre of turning(circling)
S	ship's centre of yaw motion
P	Pivot Point
V(m/s)	sway speed of S
X_p(m)	distance to P from S
r(rad/s)	yaw Speed
L	ship length
B	breadth
d	draft
R	turning circle radius
θ	drift angle, yaw angle
T	turning moment
I	moment of inertia
I_Z	polar moment of inertia
I_{XX}	I about x-axis
I_{YY}	I about y-axix
F_R	force at rudder location
a	linear acceleration
α	angular acceleration
ρ	density
Δ	displacement
G	centre of gravity
GG_1	length from G to G_1
GP	length from G to P
GR	length from G to Rudder